

Factor Quadratic Expressions of the Form $ax^2 + bx + c$

Lesson objectives

- I know how to common factor before factoring a trinomial
- I know that I need to find the product of $(a)(c)$
- I can factor a quadratic using a chart
- I can factor a quadratic using decomposition

1.1

Lesson objectives

Teachers' notes

Lesson notes

MHR Page 246 #s 2ace, 3bdf, 4ace, 5bdf,
6ace, 7bdf, 8, 9, 12 & 13

Factoring Using a Chart

1. Check your trinomial to see if you can common factor first.
2. Write the x^2 term in position #1. Write the constant term in position #4
3. Multiply the coefficient of x^2 and the constant term.

You need to find two numbers that multiply to make $(a)(c)$ and add to make b (the coefficient of x).

4. The two numbers you get in step 3 become your coefficients of x for position #2 and #3 in your chart.
5. Common factor horizontally twice and common factor vertically twice. You will now have your factors.

	spot 1	spot 3
	spot 2	spot 4

Example: $y = 3x^2 + 8x + 4$

Step 1: There are no common factors

Step 2:

	$3x^2$	
		4

Factors
 1×12
 2×6
 3×4

Step 3: $a = 3$, $c = 4$ therefore $ac = (3)(4)$

$$\frac{2 + 6}{2 \times 6} = b = 8$$

$$\frac{2 \times 6}{2 \times 6} = ac = 12$$

Step 4:

	$3x^2$	$2x$
	$6x$	4

Step 5:

		$3x$	2
x	$3x^2$	$2x$	
2	$6x$	4	

Therefore, factored form is $y = (x+2)(3x+2)$

Example: $y = 2x^2 + 11x + 12$

Step 1 Common Factor: *none*

Step 2:

	$2x^2$	
		12

Factors
 1×24
 2×12
 3×8
 4×6

Step 3: $a = 2$, $c = 12$ therefore $ac = (2)(12) = 24$

$$\frac{3 + 8}{3 \times 8} = b = 11$$

$$\frac{3 \times 8}{3 \times 8} = ac = 24$$

Step 4 Set up:

x	$2x^2$	$3x$
4	$8x$	12

Step 5 Factor:

	$2x$	3
x	$2x^2$	$3x$
4	$8x$	12

Therefore, factored form is $y = (x + 4)(2x + 3)$

Example: $y = 6x^2 - 10x - 4$

Step 1 Common Factor: $2 \Rightarrow 2(3x^2 - 5x - 2)$

Step 2:

	$3x^2$	
		-2

Factors

$1x - 6$
$2x - 3$

Step 3: $a = 3$, $c = -2$ therefore $ac = (3)(-2) = -6$

$$\frac{1}{1} + \frac{-6}{-6} = b = -5$$

$$\frac{1}{1}x \frac{-6}{-6} = ac = -6$$

Step 4 Set up:

x	$3x^2$	x
-2	$-6x$	-2

Step 5 Factor:

	$3x$	1
x	$3x^2$	x
-2	$-6x$	-2

Therefore, factored form is $y = 2(3x + 1)(x - 2)$

Example: $y = 4x^2 - 16x + 12$

Step 1 Common Factor: $4 \Rightarrow 4(x^2 - 4x + 3)$

Step 2:

	x^2	
		3

Factors

$-1x - 3$

Step 3: $a = 1$, $c = 3$ therefore $ac = (1)(3) = 3$

$$\frac{-1}{-1} + \frac{-3}{-3} = b = -4$$

$$\frac{-1}{-1}x \frac{-3}{-3} = ac = 3$$

Step 4 Set up:

x	x^2	$-x$
-3	$-3x$	3

Step 5 Factor:

	x	-1
x	x^2	$-x$
-3	$-3x$	3

Therefore, factored form is $y = 4(x - 1)(x - 3)$

The Algebra of Decomposition

Decomposition is the purely algebraic method of factoring trinomials. It starts with the same steps as the chart method:

- 1: Determine two numbers that **add** to "b" and **multiply** to the product "ac".
- 2: Decompose the **middle term**. In other words, use the two numbers you found in step one as the coefficients of x and rewrite x as two terms.
- 3: Group the **first two terms** and the **last two terms**.
- 4: Find the **common factor** of the first two terms. Find the **common factor** of the second two terms. Hint: (what is left after the common factor is removed from each group should be the same)
- 5: Common factor the **two terms** you now have.
- 6: You now have **factored form**.

Example: $y = 3x^2 + 8x + 5$

$a = 3$, $c = 5$ therefore $ac = (3)(5)$

$$\underline{3} + \underline{5} = b = 8$$

$$\underline{3} \times \underline{5} = ac = 15$$

Factors
 1×15
 3×5

$$y = 3x^2 + 3x + 5x + 5$$

$$y = (3x^2 + 3x) + (5x + 5)$$

$$y = 3x(x+1) + 5(x+1) \quad \text{**notice how both brackets are the same}$$

$$y = (x+1) \left[\frac{3x(x+1)}{(x+1)} + \frac{5(x+1)}{(x+1)} \right]$$

$$y = (x+1) \left[\frac{3x\cancel{(x+1)}}{\cancel{(x+1)}} + \frac{5\cancel{(x+1)}}{\cancel{(x+1)}} \right]$$

$$y = (x+1)(3x+5)$$

Example: $y = 2x^2 - x - 6$

$a = 2$, $c = -6$ therefore $ac = (2)(-6)$

$\underline{3} + \underline{-4} = b = -1$

$\underline{3} \times \underline{-4} = ac = -12$

$$y = 2x^2 + 3x - 4x - 6$$

$$y = x(2x + 3) - 2(2x + 3)$$

$$y = (2x + 3)(x - 2)$$

Factors

1×-12

2×-6

3×-4

Example: $y = 5x^2 + 7x + 2$

$a = 5$, $c = 2$ therefore $ac = (5)(2)$

$\underline{2} + \underline{5} = b = 7$

$\underline{2} \times \underline{5} = ac = 10$

$$y = 5x^2 + 2x + 5x + 2$$

$$y = x(5x + 2) + 1(5x + 2)$$

$$y = (5x + 2)(x + 1)$$

Factors

1×10

2×5

Example: $y = -2x^2 + 3x + 5$

Common factor: $-1 \Rightarrow -1(2x^2 - 3x - 5)$

$a = 2$, $c = -5$ therefore $ac = (2)(-5)$

$\underline{2} + \underline{-5} = b = -3$

$\underline{2} \ x - \underline{5} = ac = -10$

Factors

$1 \ x - 10$

$2 \ x - 5$

$$y = -[2x^2 - 3x - 5]$$

$$y = -[2x^2 + 2x - 5x - 5]$$

$$y = -[2x(x+1) - 5(x+1)]$$

$$y = -(x+1)(2x-5)$$